

Race to the Top: Credit Rating Bias from Competition*

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Abstract

Empirical studies have found that competition among credit-rating agencies (CRAs) deteriorates the quality of ratings. We provide a game theoretical framework to analyze CRA competition in the context of conflict of interest. We show that conflict of interest distorts the rating: the rating inflation increases as the publication fee offered by the issuer increases. As the degree of competition increases, the rating deviates even more and investors' utility declines. If the CRAs can publish unsolicited ratings, the rating inflation decreases with the degree of competition, and investors' utility improves over the solicited system. If the CRAs are paid by a third party, all CRAs truthfully report their ratings and investors' utility further improves.

Keywords: information bias, competition, credit rating shopping, conflict of interest, solicited rating, unsolicited rating.

JEL Classification:

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1 Introduction

Since the U.S. Securities and Exchange Commission introduced the registration of nationally recognized statistical rating organizations (NRSROs) in the 1970's, the credit-rating market has been a natural oligopoly. The three major credit-rating agencies account for more than 90% of the market (OECD, 2010). The U.S. credit-rating industry remained highly concentrated even after a series of reforms triggered by the Great Recession. In this paper, we explain why more competition among credit-rating agencies (thereafter CRAs) leads to rating inflation, especially for credit ratings associated with structured securities.

As the underlying assets of structured securities have become more complex, investors have had to rely more on credit ratings to infer the quality of the assets. The issuer-pay business model, in use since the 1970's (Purda, 2011), has created conflict of interest that has led to higher ratings compared to the investor-pay business model (Xia, 2011). When the issuance of structured securities were concentrated among a few top investment companies, which was the case before the Great Recession, bargaining power shifted from CRAs to the security issuer. The ability of major issuers to cherry-pick the ratings further exacerbated the CRAs' incentive to inflate the rating. Empirical evidence shows that large issuers were more likely to shop for and publish better ratings than smaller issuers (He et al., 2011).

We model CRA competition in the context of the conflict of interest inherited from the issue-pay business model. In our game theoretical framework, a monopolistic security issuer simultaneously solicits ratings from multiple CRAs. The issuer allocates a fixed budget that includes relatively low information fees paid to each CRA for an indicative rating and an additional publication fee paid to the CRA for the highest indicative rating. Upon knowing the information fee, each CRA decides whether to produce an indicative rating. If so, it acquires a noisy signal of the bond's expected return at a certain cost. Each CRA faces a trade-off between a higher profit when the issuer selects an inflated rating and the reputation cost from the discrepancy between the rating and the actual asset value.

Under the solicited rating system where the issuer only selects the highest rating for publication, only a monopolist CRA would always truthfully report the rating. Credit-rating inflation occurs where there is competition. A typical CRA in an oligopolistic market inflates the rating over a larger range of signals and to a greater degree in equilibrium as the issuer pays a higher publication fee and as more CRAs compete for the publication

fee. As a result, investors benefit from observing credit ratings because the CRAs have technology advantages and economy of scale, which allow them to acquire more precise signals. Nevertheless, investors' utility worsens as the credit-rating market becomes more competitive.

In contrast, competition among CRAs helps to deflate ratings in an unsolicited credit-rating system where CRAs are allowed to publish their ratings regardless of the issuer's selection. In equilibrium, rating inflation declines as the degree of competition increases. The reason is that under the unsolicited-rating system, producing and publishing an inflated rating would incur a reputation loss with probability one, whereas the probability of receiving the issuer's publication fee payment decreases as more CRAs operate on the market. Investors' utility improves because their observation of ratings is no longer subject to the issuer's selection; the more observed ratings, the lower the decision risks. Furthermore, under the public-funded credit-rating system in which a third party pays for the ratings, there is no conflict of interest. All CRAs truthfully report their ratings and investors' utility reaches its maximal level among the three market designs.

Our paper provides a game theoretical model for the empirical findings in the growing literature on the credit-rating market. As the rating market became more competitive when Fitch's market share grew in early the 2000's due to three major merges and acquisitions, Moody's and Standard & Poor's inflated their ratings (Becker and Milbourn, 2011). The CRAs may even deviate from their credit model to compete with their competitors (Griffin and Tang, 2012). The published credit rating was best predicted by factors associated with shopping behaviors rather than with credit quality (Cohen, 2011). For instance, larger companies receive more AAA ratings (He et al., 2011), implying a higher bargaining power when shopping for the most favorable rating (Ashcraft et al., 2011). The CRAs that provided higher ratings than their peers were more likely to be selected to rate the issuer's next corporate bond product, and this association was strongest when the underlying asset was more complex (Kronlund, 2011). As a result, structured financial products rated by only one CRA were more likely to be downgraded ex post than those rated by multiple CRAs (Benmelech and Dlugosz, 2009; Croce et al., 2011). Nevertheless, there is evidence that the CRAs were concerned about their reputation under certain circumstances. The CRAs were less likely to inflate their rating if it was close to important rating thresholds such as investment-grade ratings and prime short-term ratings because those ratings would draw more public attention (Kraft, 2014). The CRAs were also more concerned about their reputations during an economic burst than during a boom (Croce et al., 2011).

We contribute to the theoretical literature on the credit-rating inflation (Sangiorgi et al.,

2009; Sangiorgi and Spatt, 2013; Skreta and Veldkamp, 2009) by providing a new perspective on conflict of interest as a source of rating inflation. We model conflict of interest as the strategic competition among CRAs to receive the publication fee from the issuer, whereas the literature models conflict of interest as the issuers' strategic selection of credit ratings. In our model, CRAs are the strategic players who have an incentive to inflate their ratings when the security issuer shops for ratings. The monopolistic security issuer is a non-strategic player who allocates its budget between the information fee and the publication fee, while simultaneously soliciting indicative ratings, and always publishes the highest rating. In the existing models, the issuer is the strategic player who sequentially solicits indicative ratings and stops shopping when the current one reaches the highest expectation level; the CRAs always truthfully report the noisy signal they observe. In both our model and the previous models, investors update the true return of the bond based on the published rating and offer a price for the security.

We add competition to the literature on unsolicited ratings. Fulghieri et al. (2013) model unsolicited ratings as a monopolistic CRA's "threat" to the security issuer in a two-period incomplete information game. The potential to publish an unfavorable rating regardless of the issuer's solicitation increases the CRA's bargaining power and mitigates the conflict of interest (Sangiorgi et al., 2009; Fulghieri et al., 2013). Our model extends the unsolicited rating system to multiple CRAs compete for the publication fee from the security issuer. The unsolicited ratings eliminate the issuer's selection effect so that each participating CRA will realize a reputation loss by providing an inflated rating. Therefore, competition alleviates the CRAs' incentive to inflate the rating in the solicited rating system even though higher publication fees still lead to more inflation due to the conflict of interest. Greater competition leads to lower rating inflation because the chance of winning the publication fee decreases as the number of CRAs increases, yet there is surely a reputation cost. The reputation disciplines the CRAs (Elamin, 2014, 2012; Mathis et al., 2009; Stolper, 2009; Frenkel, 2010; Manso, 2013; Fulghieri et al., 2013; Bolton et al., 2012) more effectively under an unsolicited-rating system than under a solicited-rating system.

Our work is built upon the information disclosure literature (Bolton et al., 2007; Lizzeri, 1999; Damiano et al., 2008; Albano and Lizzeri, 2001) in which information manipulation is examined under different market structures and levels of competition. We add to the literature by considering competition in the context of conflict of interest. When the security issuer ("payer") pays for the rating received by investors ("receiver"), CRAs strategically provide indicative ratings to maximize their profits rather than simply truthfully reporting their updated states. Therefore, we find that competition increases the degree of rating in-

flation and worsens the market investors' decision quality. Our result is contrary to Lizzeri (1999) and Bolton et al. (2007), who found that competition leads to full information disclosure when the payer and the receiver are the same agent. We further show that the CRAs always truthfully reports their ratings and investors' utility is the highest when the ratings are paid for by a third-party, in which case the conflict of interest is eliminated.

The remainder of the paper is organized as follows: Section 2 introduces the model, strategies, and equilibrium concept. Section 3 provides an equilibrium characterization under the solicited rating system. We discuss the degree of rating inflation with respect to the publication fee, and the level of competition among CRAs. Then we show how upward biased ratings can worsen investors' decision quality as well as social welfare. Section 4 discusses the credit-rating market and unsolicited ratings. We show that, as long as the issuer remains the primary income source for the CRAs, allowing the CRAs to publish unsolicited ratings may reduce, but not fully eliminate, the information distortion that results from rating inflation. Section 5 concludes.

2 The Model

We begin by describing a static model with a single bond issuer, n CRAs, and a continuum of investors who interact in the market for structured securities. All players are risk-neutral. The bond issuer intends to sell a one-period structured security to investors. The return of the asset at maturity is a random variable X , the realization of which is determined by Nature. The random variable follows a Normal Distribution $X \sim N(\mu_X, \sigma_X^2)$, which is the common prior belief known to all players.

We denote each of the credit-rating agencies as $\text{CRA}_i, i = 1, \dots, n$. In addition to prior knowledge of the asset's return, each CRA may invest in an investigation process to acquire a private signal $S_i = s_i$ regarding the possible realization of random variable X . The noisy signal is a random variable $S_i = X + \epsilon_i$, where $\epsilon_i \sim N(0, \sigma_i^2)$, $\text{corr}(\epsilon_i, \epsilon_j) = 0$ for all $i \neq j$, and $\text{corr}(\epsilon_i, X) = 0$ for all $i = 1, \dots, n$.

Moreover, the investigation process is costly: CRA_i needs to pay $c_i(\sigma_i)$ to obtain a noisy signal s_i with information precision σ_i . Each CRA_i chooses the precision $\sigma_i \in [0, \infty)$. We assume that the cost function decreases in the variance of the disturbance term: $c'_i(\sigma_i) < 0$. Namely, the more precise a signal CRA_i acquires, the more it must pay. Upon observing a noisy signal s_i , CRA_i produces an indicative rating $a_i \in \mathbb{R}$. If CRA_i does not acquire a

private signal, it remains uninformed and cannot produce an indicative rating.

Investors also observe a noisy signal s_0 regarding the asset return X . The precision of investors' signal is coarser than any signal acquired by a typical CRA_i due to CRAs' technology advantage and economy of scale. Conditional on the realization $X = x$, we denote $s_0 = x + \epsilon_0$, $\epsilon_0 \sim N(0, \sigma_0^2)$, $\text{corr}(\epsilon_0, \epsilon_i) = 0$ for all i , $\text{corr}(\epsilon, X) = 0$, and $\sigma_i < \sigma_0$. The above-specified information structures are known to all players, as are all higher levels of knowledge.

The credit-rating game consists of five stages. At *stage 1*, the issuer allocates a fixed budget of T between an information fee λ and a publication fee χ ,¹ and the issuer announces the payment scheme to all CRAs. The information fee λ is a payment to any CRA which produces an indicative rating, and the publication fee χ is a payment only to the CRA whose rating is chosen to be published. A behavior strategy of the issuer $(\lambda, \chi) \in \mathbb{R}_+^2$ specifies a vector of fees subject to the fixed budget $\chi + m \cdot \lambda = T$, where m is the number of CRAs that choose to conduct the costly investigation and acquire a noisy signal s_i to produce a rating for the issuer's consideration.

At *stage 2*, all CRAs observe the issuer's choice of the payment scheme (λ, χ) . Each CRA decides whether to invest in an investigation for a noisy signal or opt out. Upon having determined to acquire a signal s_i , the CRA further determines the information precision $\sigma_i \in \mathbb{R}$ of the investigation process. The behavior strategy of a typical CRA at this stage is described by the function

$$\beta_i : \mathbb{R}_+^2 \rightarrow \Delta\{\text{Acquire, Opt Out}\} \times \Delta\mathbb{R}$$

as a mapping from the issuer's strategy space to the Cartesian product of the family of probability distributions over the set of information acquisition options and the information precision.

At *stage 3*, each CRA that has chosen to acquire costly information observes a noisy signal $s_i \in \mathbb{R}$ and proposes an indicative rating $a_i \in \mathbb{R}$. We assume that a CRA commits to publishing its indicative rating if being selected by the issuer later. A typical CRA's behav-

¹We assume that the fixed budget T is sufficient to cover all CRAs' potential information costs plus an adjustment term where F is the CDF of random variable S_i :

$$T \geq \frac{\sigma_X^2}{\sigma^2 + \sigma_X^2} \cdot \frac{\int_{-\infty}^{\mu_X + 3\sqrt{\sigma_X^2 + \sigma^2}} [F(t)]^{n-1} dt}{[F(\mu_X + 3\sqrt{\sigma_X^2 + \sigma^2})]^{n-1}} + n \cdot c$$

ior strategy at this stage is a mapping from the signal realization space to the probability distribution over the set of possible ratings:

$$\alpha_i : \mathbb{R} \rightarrow \Delta\mathbb{R}$$

At *stage 4*, upon observing all the indicative ratings from all participating CRAs, the issuer selects a rating $\bar{a} \in \{a_1, \dots, a_m\}$ s.t. $\bar{a} \geq a_j, \forall j$, pays the CRA that produces the rating \bar{a} , and publishes the selected rating to all investors. If no CRA produces a rating, the issuer publishes none.

The issuer's selection results in three potential levels of a CRAs' payoffs, depending on whether a CRA participates in producing a rating and whether its rating is chosen. In the first case, a CRA opts out without the information acquisition. We normalize a CRA's payoff to 0 in this case. Now consider a typical agency CRA_{*i*} that spends a cost c_i at the information acquisition stage and produces an indicative rating $a_i \in \mathbb{R}$. In this second case, this indicative rating a_i is not selected by the issuer in the end, CRA_{*i*} receives the information fee λ from the issuer and has a payoff of $\lambda - c_i(\sigma)$. No future event will alter the CRA's payment. In the third case, the issuer selects the indicative rating a_i , the CRA_{*i*} receives the publication fee χ and the information fee λ . When the asset matures and the true value x is realized, this CRA also suffers a reputation loss from the market's perception of a possibly inaccurate rating.

We assume the loss is of the form $-|a_i - x|$.² We denote the positive term $|a_i - x|$, i.e. the absolute value of discrepancy between the rating and the actual bond quality at maturity, as the CRA's *reputation cost*. Reputation cost increases as the discrepancy between the published rating a_i and the realized return x expands. We represent the reputation cost by an absolute-value function, which indicates that both over-rating and under-rating will induce a reputation loss. Assume that the discount factor is 1 between stages, a winning CRA_{*i*}'s payoff is

$$\lambda + \chi - c_i(\sigma) - |a_i - x|$$

It is worth mentioning that a straightforward report of the noisy signal s_i may also

²Reputation has been discussed mostly in discrete cases where the issuer either default or pays back the bond at maturity. Bolton et al. (2012) and Elamin (2012) model the reputation cost as the loss of the CRA's entire future business once it lies. Such construct of reputation cost requires investors to perfectly identify whether the CRA lies in the event of default, which is unlikely in the real world. Mathis et al. (2009) and Fulghieri et al. (2013) model the CRA's reputation as a posterior probability that investors and issuers assign to the event that the CRA is truthful.

induce a reputation cost for the CRA. This is because CRAs only observe a signal s_i subject to a disturbance term ϵ_i . In reality, investors cannot distinguish a deliberate misreport from a CRA’s inaccurate estimations, the “quality” of a CRA’s ratings is evaluated in light of the deviation of the realized return of the asset at maturity.

At *stage 5*, the issuer sells the structured security to the market, and investors make their asset purchase decisions. When the instrument matures, the realization $X = x$ is revealed to all players, and payoffs are realized. We assume investors have a utility function

$$V = E(X|I) - r \cdot Var(X|I)$$

where I is the information set and r is the risk premium. In the absence of a CRA’s rating, $I = s_0$, investors’ raw data of the random variable’s realization. When a rating \bar{a} is published, $I = \bar{a}$.

We shall focus on Perfect Bayesian Equilibria of the credit-rating game in which all players use symmetric pure strategies. In a Perfect Bayesian equilibrium, each CRA maximizes its expected profit, and the issuer reaches its budget–balance condition. We restrict our attention to the CRAs’ symmetric equilibrium strategy profiles in which they choose the same information precision $\sigma_i = \sigma$ upon having decided to acquire information and to produce an indicative rating. The incurred information cost becomes homogeneous: $c_i(\sigma_i) = c(\sigma) = c$ for all i . Depending on the value of c , the number of participating CRAs from Stage 3 on is either $m = 0$ or $m = n$.

3 The Baseline: Solicited Rating System

In this section we discuss the solicited rating system in which all participating CRAs provide indicative ratings and the issuer selects only one rating for final publication. We first characterize the CRAs’ ratings in a symmetric Perfect Bayesian equilibrium. Comparative static results show that the degree of rating inflation increases with the issuer’s publication fee χ and the number of rating agencies n . Then we evaluate investors’ utility and social welfare in the presence of rating inflation. We derive conditions under which CRAs’ costly information acquisition and rating production may become socially undesirable.

3.1 Rating Inflation: The Negative Impact of Competition and Conflict of Interest

In this subsection, we characterize the CRAs' rating strategy in equilibrium. The theoretical results indicate that rating inflation increases as the publication fee increases and as the market becomes more competitive. In subsequent parts, we examine investors' utility and social welfare.

Each CRA maximizes its expected payoff by choosing the value of the indicative rating, taking the issuer's information fee λ and publication fee χ as given. As stated above, we focus on the symmetric Perfect Bayesian equilibrium in which the CRAs either all acquire noisy signals with the precision $\sigma_i = \sigma$ or all opt out. When they opt out, the issuer has no indicative rating to select from, and investors make their purchase decisions on their own signal s_0 . We now focus on the symmetric equilibrium where all CRAs participate in the rating. In this case, $m = n$ CRAs provide indicative ratings, upon each observing a noisy signal s_i at a cost c .

Given each CRA's noisy signal observation s_i , we define a CRA's rating strategy $\alpha_i(s_i)$ as a *truthful report* if

$$\alpha_i(s_i) = E(X|s_i) = \frac{\mu_X \cdot \sigma^2 + s_i \cdot \sigma_X^2}{\sigma^2 + \sigma_X^2}.$$

where $E(X|s_i)$ is the expected value of the asset's return conditional on a signal s_i randomly generated from the distribution $S_i \sim N(\mu_X, \sigma^2 + \sigma_X^2)$. Any rating other than $E(X|s_i)$ upon observing s_i is a *non-truthful report*. Note that *truthfully reporting* does not mean a CRA precisely predicts the actual realization of the asset return x with probability 1. In reality, information intermediaries can only perform analyses and forecasts to provide their "best educated guess" for the asset value. The high volatility of the financial market makes it impossible for an intermediary to observe the actual asset value without disturbances. Thus, any rating is subject to inaccuracy even if it is a truthful report of the CRA's estimation regarding the structured product in question.

A CRA_{*i*} who participates in the rating game will maximize its expected payoff with the form

$$\begin{aligned} E\pi &= [\lambda + \chi - c - |a_i - x|s_i|] \cdot Prob(win) + [\lambda - c] \cdot [1 - Prob(win)] \\ &= [\chi - E_{X|S_i}|a_i - x|s_i|] \cdot Prob(win) + [\lambda - c] \end{aligned}$$

where $Prob(win)$ denotes the probability that CRA_i wins the issuer's selection process and publishes its indicative rating. Given the symmetry of $\alpha(s_i)$, $Prob(win) = Prob(a_i \geq a_j, i \neq j) = [F(\alpha^{-1}(a_i))]^{n-1}$. Hence, the CRA_i 's optimization problem reduces to:

$$\max_{a_i} [\chi - E_{X|s_i}[a_i - x|s_i]] \cdot Prob(a_i \geq a_j, i \neq j) + [\lambda - c]$$

Solving the maximization problem, we have the CRA 's optimal rating strategy characterized by the following proposition:

Proposition 1. *If all CRA s obtain noisy signals and produce indicative ratings, each CRA follows a symmetric and monotone rating strategy. Given a noisy signal s_i received by CRA_i , the optimal rating strategy is*

$$\alpha(s_i) = \begin{cases} \tilde{a}(s_i), & \text{if } \tilde{a}(s_i) > E(X|s_i); \\ E(X|s_i), & \text{otherwise.} \end{cases}$$

where

$$\tilde{a}(s_i) = \frac{\mu_X \cdot \sigma^2 + s_i \cdot \sigma_X^2}{\sigma^2 + \sigma_X^2} + \chi - \frac{\sigma_X^2}{\sigma^2 + \sigma_X^2} \cdot \frac{\int_{-\infty}^{s_i} [F(t)]^{n-1} dt}{[F(s_i)]^{n-1}},$$

This optimal rating strategy is a strictly increasing function of the noisy signal s_i .

The proof for this proposition is included in Appendix A.1. This result indicates that a participating CRA 's equilibrium rating strategy is a piecewise function. Note that $E(X|s_i) = \frac{\mu_X \cdot \sigma^2 + s_i \cdot \sigma_X^2}{\sigma^2 + \sigma_X^2}$, therefore, when $\chi - \frac{\sigma_X^2}{\sigma^2 + \sigma_X^2} \cdot \frac{\int_{-\infty}^{s_i} [F(t)]^{n-1} dt}{[F(s_i)]^{n-1}} > 0$, CRA_i 's optimal reporting rule is $\tilde{a}(s_i)$; otherwise CRA_i follows a *truthful reporting* strategy and provides an indicative rating equal to the conditional expected asset return $E(X|s_i)$.

Note that the *non-truthful report* function $\tilde{a}(s_i)$ consists of three parts. The first component $\frac{\mu_X \cdot \sigma^2 + s_i \cdot \sigma_X^2}{\sigma^2 + \sigma_X^2} = E(X|s_i)$ is the expected value of the asset conditional on the noisy signal observation s_i . The second element is the publication fee, χ , as the CRA 's future income promised by the issuer. The third negative part is an adjustment term as a function of the value of s_i and the number of participating CRA s, n . Given an s_i , the value of this term declines as n increases.

It is worth noting that the optimal rating strategy reduces to the *truthful report* strategy when $n = 1$, i.e., there is a monopolist CRA ,

Corollary 1. *When there is a monopolistic CRA operating on the market, the monopolistic*

CRA adopts an optimal rating strategy

$$\alpha(s_i) = E(X|s_i) = \frac{\mu_X \cdot \sigma^2 + s_i \cdot \sigma_X^2}{\sigma^2 + \sigma_X^2}.$$

The corollary directly follows the derivation of Proposition 1 when setting $n = 1$. No rating inflation appears when there is only one CRA producing an indicative rating for the issuer. In this situation, the probability of being selected, $Prob(win)$, equals one. When there is no uncertainty regarding receipt of receiving the publication fee, the monopolist CRA reports its estimation truthfully to minimize the potential reputation loss.

Next, we discuss the issuer's choice of a payment scheme. Specifically, we provide a condition for the issuers' choice of information fee λ and publication fee χ that could incentivize the CRAs to participate in the rating. All CRAs would opt out and produce no rating if the issuer failed to provide an appropriate incentive scheme.

In the symmetric Perfect Bayesian equilibrium under discussion, the issuer chooses the payment scheme to ensure that the participating CRAs' expected payoff is greater than zero, the payoff of opting out the rating game. Particularly, the issuer's choice of (λ, χ) satisfies the **Participation Constraint (PC)** and **Budget Constraint (BC)** as the following:

$$\begin{aligned} \mathbf{PC} \text{ for CRAs : } & \quad [\chi - E_{X|S_i}[a_i - x|s_i]] \cdot Prob(win) + [\lambda - c] \geq 0, \forall s_i \\ \mathbf{BC} \text{ of the Issuer : } & \quad n \cdot \lambda + \chi = T \end{aligned}$$

The above condition can be further simplified. We denote the order statistics of all CRAs' noisy signals as $s_{(1)} < \dots < s_{(n)}$. Proposition 1 shows that both CRAs' optimal rating strategies $\alpha(s_i)$ and equilibrium probability of winning $Prob(win)$ are monotone increasing in $s_{(i)}$, $\forall i = 1, \dots, n$. Thus, the expected payoff for a participating CRA is at its lowest when $S_i = s_{(1)}$. It is also easy to check, as in this situation $Prob(win|S_i = s_{(1)}) < 1/n$. Furthermore, we can rewrite the issuer's budget constraint as $\chi = T - n \cdot \lambda$. Thus, the **PC** and the **BC** conditions for issuer's choice of (λ, χ) reduce to the following equation:

$$\lambda \geq \frac{-(T - E_{X|S_i}[a_i - x|s_i]) \cdot Prob(win|S_i = s_{(1)}) + c}{1 - n \cdot Prob(win|S_i = s_{(1)})} \quad (1)$$

The issuer's choice of $\lambda \in \mathbb{R}$ satisfying equation 1 sustains the symmetric Perfect Bayesian equilibrium of the credit-rating game. Denote the value of information fee $\underline{\lambda}$ when the equality holds. Note that when $s_{(1)} \rightarrow \mu_X - 3\sqrt{\sigma_X^2 + \sigma^2}$, i.e., the lower bond of the 99.87%

confidence interval, $Prob(win|S_i = s_{(1)}) \rightarrow 0$, and $\underline{\lambda} \rightarrow c$. As long as the information fee can cover its information cost, even the CRA with the lowest signal realization has an incentive to participate. In this situation, a symmetric Perfect Bayesian equilibrium exists, and the issuer achieves the budget-balance condition.

3.1.1 Rating Inflation from a Conflict of Interest

This subsection discusses the relation between CRAs' rating strategies as a result of bias from the issuer's selection. Specifically, we investigate how the degree of *rating inflation* varies with respect to the issuer's publication fee χ .

Definition 1. *The **rating inflation** of CRA_i is defined as the distance between the indicative rating and expected value of X the conditional on a noisy signal s_i :*

$$\alpha(s_i) - E(X|s_i)$$

In other words, the notion of *rating inflation* measures the degree to which CRA_i 's indicative rating exceeds the *truthful report* $E(X|s_i)$. Proposition 1 indicates that CRA_i follows the non-truthful report strategy $\tilde{a}(s_i)$ whenever $\tilde{a}(s_i) \geq E(X|s_i)$. Thus it is useful to first present two properties of the function $\tilde{a}(s_i)$.

Lemma 1. *$\tilde{a}(s_i)$ is concave within the range $[\mu_X - 3\sqrt{\sigma_X^2 + \sigma^2}, \mu_X + 3\sqrt{\sigma_X^2 + \sigma^2}]$ and $\tilde{a}'(s_i) \leq \frac{dE(X|s_i)}{ds_i}$. Moreover, $\lim_{s_i \rightarrow +\infty} \tilde{a}'(s_i) = 0$*

The proof of this lemma is included in Appendix A.2. Note that we focus on the interval $s_i \in [\mu_X - 3\sqrt{\sigma_X^2 + \sigma^2}, \mu_X + 3\sqrt{\sigma_X^2 + \sigma^2}]$, the 99.87% confidence interval, where μ_X and $\sqrt{\sigma_X^2 + \sigma^2}$ are the mean and the standard deviation, respectively, of the noisy signal s_i . Lemma 1 indicates that the function $\tilde{a}(s_i)$ is always flatter than $E(X|s_i)$, and $\tilde{a}'(s_i)$ decreases to 0 as s_i grows. As a result, $\tilde{a}(s_i)$ either intersects with $E(X|s_i)$ at most once or always lies below it. Since we focus on the interval within three standard deviations, the only case in which the two curves intersect is as Figure 2 shows: the function $\tilde{a}(s_i)$ exceeds $E(X|s_i)$ when S_i 's realization s_i is relatively small, and $\tilde{a}(s_i)$ lies below $E(X|s_i)$ if the realization s_i is extremely large.

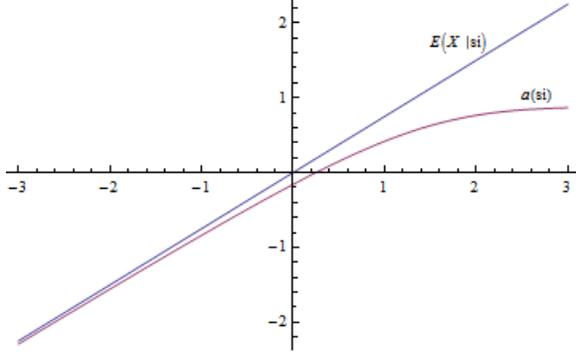


Figure 1: Non-Inflated Ratings

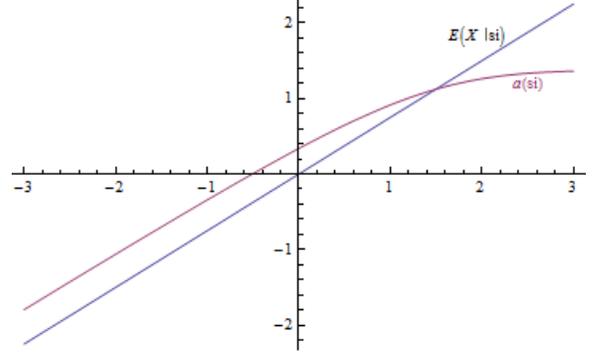


Figure 2: Partially Inflated Ratings

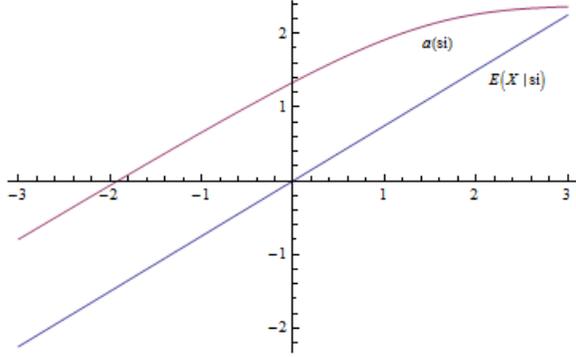


Figure 3: Inflated Ratings

Notes: Parameter values: $\mu_X = 0$, $\sqrt{\sigma_X^2 + \sigma^2} = 1$, $\sigma^2 = \frac{1}{4}$, $\sigma_X^2 = \frac{3}{4}$. $n = 6$ fixed. $\chi = \frac{1}{240}, \frac{1}{2}, \frac{3}{2}$ for Figure 1, 2, 3 respectively. The blue line represents $E(X|s_i)$ while the purple curve represents $\tilde{a}(s_i)$.

Corollary 2. *There exist two cutoff values χ_1, χ_2 of the issuer's publication fee χ :*

$$\chi_1 = \frac{\sigma_X^2}{\sigma^2 + \sigma_X^2} \cdot \frac{\int_{-\infty}^{\mu_X - 3\sqrt{\sigma_X^2 + \sigma^2}} [F(t)]^{n-1} dt}{[F(\mu_X - 3\sqrt{\sigma_X^2 + \sigma^2})]^{n-1}}$$

$$\chi_2 = \frac{\sigma_X^2}{\sigma^2 + \sigma_X^2} \cdot \frac{\int_{-\infty}^{\mu_X + 3\sqrt{\sigma_X^2 + \sigma^2}} [F(t)]^{n-1} dt}{[F(\mu_X + 3\sqrt{\sigma_X^2 + \sigma^2})]^{n-1}}$$

where $F(\cdot)$ denotes the CDF of a normal distribution with mean μ_X and variance $\sigma^2 + \sigma_X^2$. The rating inflation of CRA_i depends on the following conditions:

- if $\chi \leq \chi_1$, the credit ratings are not inflated, i.e., every CRA truthfully reports $E(X|s_i)$;
- if $\chi \in [\chi_1, \chi_2]$, the credit ratings are partially inflated, i.e., each CRA adopts reporting rule $\tilde{a}(s_i)$ when the noisy signal s_i is not large, while each CRA truthfully reports $E(X|s_i)$ when s_i relatively large; and
- if $\chi \geq \chi_2$, the credit ratings are always inflated, i.e., each CRA follows the reporting

rule $\tilde{a}(s_i)$.

These conditions are derived for a given level of competition n^3 . Figures 1, 2, 3 depict the three cases of rating inflation pattern specified by Corollary 2. The issuer's choice of χ provides the CRAs with incentives to compete for the credit selection. The corollary reveals that rating inflation increases as the issuer's publication fee χ increases. The gap between the rating function $\tilde{a}(s_i)$ and the truthful report $E(X|s_i)$ expands. Moreover, rating inflation exists for a wider range of s_i , which indicates that CRAs are more likely to inflate their indicative ratings for some possible realizations of noisy signal s_i .

3.1.2 Rating Inflation from Competition

In this subsection we present results on the relation between rating inflation and competition among the CRAs. We derive the comparative statics of each CRA's rating strategy with respect to the number of participating CRAs, fixing the issuer's choice of publication fee χ .

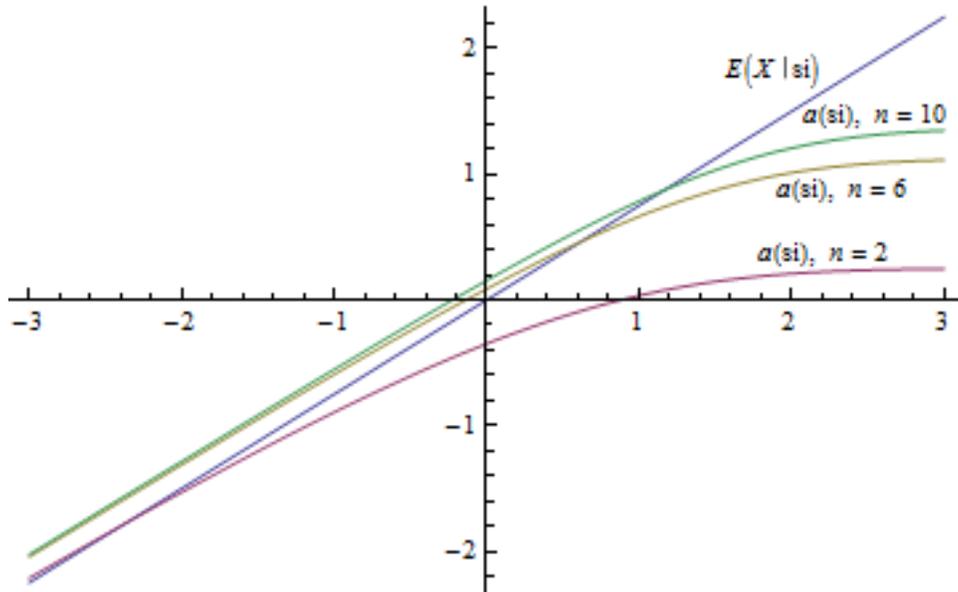


Figure 4: Comparative Statics w.r.t. n

Notes: Parameter values: $\mu_X = 0$, $\sqrt{\sigma_X^2 + \sigma^2} = 1$, $\sigma^2 = \frac{1}{4}$, $\sigma_X^2 = \frac{3}{4}$. $\chi = \frac{1}{16}$ fixed. The blue line is depicted for $E(X|s_i)$ while the purple, yellow, and green curves are for $\tilde{a}(s_i)$ when $n = 2, 6, 10$ respectively.

Corollary 3. *When the publication fee is fixed at χ , a CRA inflates its indicative rating more as the number of participating CRAs, n , increases.*

³Note that the **BC** and **PC** conditions imply the upper bound of the publication fee, $\bar{\chi} = T - n \cdot \underline{\lambda}$. It is easy to check that, given the model setup, $\bar{\chi} > \chi_2$. Thus the **PC** and **BC** conditions are not binding and all the three cases discussed in Corollary 2 are possible.

The corollary results from the fact that a CRA_i 's non-truthful rating strategy, $\tilde{a}(s_i)$ shifts upwards when the number of CRAs, n , increases, given each chosen value of χ . Figure 4 depicts this comparative static result for a fixed χ value. As n becomes larger, the function $\tilde{a}(s_i)$ moves up, which results in rating inflation over a larger range of signal s_i . Meanwhile, the gap between $\tilde{a}(s_i)$ and the truthful report strategy $E(X|s_i)$ widens. The degree of rating inflation is augmented by the degree of competition in the credit-rating market. In summary, the issuer's selection of a rating encourages the CRAs to inflate their ratings as long as the information costs and reputation losses from inaccurate ratings can be compensated for by the expected payoff from issuer's publication fee χ . As the credit market becomes more competitive, the degree of rating inflation also rises.

3.2 Investors' Utility: The Negative Impact of Competition

This section examines the impact of CRA competition on investors' welfare. We compare the investors' utilities in the following situations: purchasing the structured financial product (1) with no credit rating, (2) with one credit rating from a monopolistic CRA, and (3) with one credit rating selected by the issuer. We show that CRAs do provide investors with incremental information. Meanwhile, when compared to cases with one monopolistic CRA, the issuer's selection among multiple CRAs would make investors strictly worse off.

We will focus on *rational but ignorant* investors throughout our discussion. Investors are *rational* in the sense that they are aware of the issuer's credit shopping and the potential rating inflation from the issuer's selection. Investors do not simply treat issuer-published credit ratings as an unbiased estimation of the conditional expected return of the asset $E(X|S_i = s_i)$. Instead, they calculate the asset's expected return $E(X|S_i = s_{(n)})$ conditional on the assumption that the observed rating is generated from the n -th order statistic of all CRAs' noisy signals.

On the other hand, investors are *ignorant* because they lack relevant information regarding the issuer's payment scheme and the degree of competition among CRAs. The interaction between the issuer and CRAs is hidden from investors. When the value of key parameters n , χ and λ is inside the "black-box" of the credit-rating market, investors cannot fully identify the asset's actual expected return conditional on observing the published rating as a result of the issuer's selection bias. In other words, the rational investors' level of sophistication is constrained by the information available to them.

As defined in Section 2, investors' utility is the expected return of the bond X minus a

risk premium adjusted for the variance of X conditional on the information I available to investors. Denote the investors' expected utility as V_0 when no credit rating is published by the issuer. In this situation, investors estimate the expected return of the asset based on the common prior and the coarse signal s_0 :

$$V_0 = E_{X|S_0}(X|s_0) - r \cdot \text{Var}_{X|S_0}(X|s_0)$$

where the random variable $S_0 \sim N(\mu_X, \sigma_0^2 + \sigma_X^2)$.

Meanwhile, denote V_1 the investors' expected utility when there is one monopolistic CRA that produces and publishes the credit rating:

$$V_1 = E_{X|S_i}(X|E(X|s_i)) - r \cdot \text{Var}_{X|S_i}(X|E(X|s_i))$$

As revealed by Corollary 1, a monopolistic CRA adopts a *truthful report* rating strategy $\alpha(s_i) = E(X|s_i), \forall s_i$. The selection bias diminishes, and there is little information distortion associated with the published rating.

Furthermore, we use V_n to represent the investors' expected utility when the issuer selects and publishes one out of n indicative ratings:

$$V_n = E_{X|S_{(n)}}(X|\alpha(s_{(n)})) - r \cdot \text{Var}_{X|S_{(n)}}(X|\alpha(s_{(n)}))$$

The investors' expected utility V_n is evaluated under the distribution of the n -th order statistic $S_{(n)}$. This is from the result that a typical CRA's rating strategy, $\alpha_i(s_i)$, is increasing in the noisy signal s_i . As the issuer always selects the most favorable indicative rating for publication, investors' observation $\alpha(s_{(n)})$ is associated with the highest noisy signal $s_{(n)}$. The following result ranks the investors' expected utilities under the three circumstances:

Proposition 2. *When compared to a market without any credit rating, investors' utility improves when the credit rating is provided by a monopolistic CRA: $V_1 > V_0$. However, the investors' expected utility decreases as the number of CRAs increases: $V_n < V_1$.*

The proof of this proposition is included in Appendix A.3. The first part echoes the argument that investors benefit from a monopolistic CRA's "incremental information" (Purda, 2011). It indicates that investors' welfare improves more with a CRA's rating than it does without such an information source. This effect stems directly from the assumption of the model that investors' prior information is coarser than that could be

acquired by a CRA at a certain cost. More precise signals reduce investors' decision risks as the conditional variance of the asset's return X decreases.

The second part of the proposition shows that investors are worse off as the credit-rating market becomes more competitive. We consider two sub-cases: $\tilde{a}(s_i) \leq E(X|s_i)$ and $\tilde{a}(s_i) > E(X|s_i)$. In the former case, all CRAs adopt the truthful report strategy, $\alpha_i(s_i) = E(X|s_i)$. Thus, V_n is calculated as the expected return of X conditional on observing a rating $E(X|s_{(n)})$. In the latter situation, rating inflation exists and V_n is derived conditional on an observed rating $\tilde{a}(s_{(n)})$. In both cases, the inequality $V_n < V_1$ holds, which indicates that the investors' utility declines as the number of CRAs, n , exceeds one.

3.3 Information Cost and Social Welfare

In this subsection we investigate the social welfare effect of the solicited rating system. We consider the same three situations as in the previous section: a market without any credit rating, with one credit rating from a monopolistic CRA, and with one credit rating selected by the issuer. We derive upper bounds for the CRAs' costs so that the information acquisition and rating production are socially optimal.

We define social welfare to be the sum of the investors' utility and CRAs' income minus CRAs' costs and the issuer's payments. We denote W_0 as the social welfare when no CRA acquires information and no credit rating is published. The issuer does not make any payment, and none of the CRAs bear either the information cost or reputation loss. Thus we have $W_0 = V_0$.

Let W_1 denote the social welfare when there is a monopolistic CRA operating on the market.

$$\begin{aligned} W_1 &= V_1 - T + (\chi + \lambda - c - E_{X|s_i}(|E(X|s_i) - X|)) \\ &= V_1 - c - E_{X|s_i}(|E(X|s_i) - X|) \\ &= V_1 - c \end{aligned}$$

where $T = \chi + \lambda$ is a transfer from the issuer to the sole CRA, and $E_{X|s_i}(|E(X|s_i) - X|) = 0$. As such, the above expression reduces to the investors' utility V_1 less CRA's information cost c .

We denote W_n as the social welfare when n CRAs acquire information and produce indicative ratings, and the published rating results from an issuer's selection procedure.

$$\begin{aligned} W_n &= V_n - T + (\chi + \lambda - c - E_{X|S(n)}(|\alpha(s(n)) - X|)) + (n - 1) \cdot (\lambda - c) \\ &= V_n - nc - E_{X|S(n)}(|\alpha(s(n)) - X|) \end{aligned}$$

where the issuer's payments to all the n CRAs are $T = \chi + n \cdot \lambda$. Thus W_n reduces to the investors' utility minus all CRAs information costs and the winning CRA's reputation loss.

The static comparison of W_n , W_1 , and W_0 depends on the CRAs' information acquisition cost c . We have the following proposition that describes the comparison.

Proposition 3. *There exists an upper bound of information cost $\bar{c}_1 = V_1 - V_0$ such that $W_1 \geq W_0$ only when $c \leq \bar{c}_1$. There exists another upper bound of information cost $\bar{c}_n = (V_n - V_0 - E_{X|S(n)}(|\alpha(s(n)) - X|))/n < \bar{c}_1$ such that $W_n \geq W_0$ only when $c \leq \bar{c}_n$.*

This proposition follows directly from the derivation of Proposition 2. The upper bounds suggest that it is socially optimal to acquire information through CRAs only when the information cost is below a certain threshold. Otherwise, the information cost completely offsets any benefit investors' would realize from the incremental information, and it is socially optimal for investors to make purchase decisions based on their prior information. Note that $\bar{c}_1 > \bar{c}_n$, which implies that the information cost needs to be sufficiently low to support a competitive market where there are information costs for multiple CRAs.

4 Unsolicited Rating and Public-Pay Systems

In this section we consider two variations of the credit rating market design. In Section 4.1, we allow the CRAs to publish unsolicited credit ratings irrespective of an issuer's selection. Our results indicate that competition among CRAs mitigates the effects of rating inflation because any inflated rating induces a reputation cost for the CRA that voluntarily provides an unsolicited rating. Nonetheless, the issuer's publication fee payment still encourages CRAs to produce inflated ratings because of a conflict of interest. In Section 4.2 we further consider a public-pay model (Deb and Murphy, 2009; Kashyap and Kovrijnykh, 2013; Faure-Grimaud et al., 2009; Cole and Cooley, 2014) which eliminates the conflict of interest. Under such system, all CRAs truthfully report their ratings and the investors' utility is the highest among all the three credit market designs.

4.1 Unsolicited Ratings, Competition, and Conflict of Interest

In the unsolicited rating system, we allow the CRAs whose indicative ratings have not been selected by the issuer to publish the rating voluntarily at *stage 4*. Therefore, a CRA faces a different optimization problem given the issuer's choice of χ and λ , which satisfy **(PC)** and **(BC)** conditions. All remaining settings of the credit-rating model remain as described in Section 2.

As before, we focus on the symmetric Perfect Bayesian equilibrium in which each CRA acquires a noisy signal with the same precision $\sigma_i = \sigma$, produces an indicative rating, and publishes the rating irrespective of the issuer's selection. Since the indicative rating would be published anyway, the expected reputation loss from a rating discrepancy, $E|a_i - x|$, occurs in the objective function regardless of whether the CRA wins the issuer's selection. Specifically, a CRA's expected payoff becomes:

$$\begin{aligned} E\pi &= [\lambda + \chi - c - |a_i - x|s_i] \cdot \text{Prob}(\text{win}) + [\lambda - c - |a_i - x|s_i] \cdot (1 - \text{Prob}(\text{win})) \\ &= \chi \cdot \text{Prob}(\text{win}) + [\lambda - c] - E_{X|S_i}|a_i - x|s_i| \end{aligned}$$

where we continue to use $\text{Prob}(\text{win})$ to denote the probability that the CRA_{*i*}'s indicative rating is selected by the issuer. We further simplify the problem using the symmetry of CRAs' rating strategies, replacing $\text{Prob}(\text{win})$ with $\text{Prob}(a_i \geq a_j, i \neq j) = [F(\alpha^{-1}(a_i))]^{n-1}$. CRA_{*i*}'s optimization problem reduces to:

$$\max_{a_i} \chi \cdot \text{Prob}(a_i \geq a_j, i \neq j) + \lambda - c - E_{X|S_i}|a_i - x|s_i|$$

We solve the maximization problem and reach the following result:

Proposition 4. *Under the system that allows publication of unsolicited ratings, in a symmetric Perfect Bayesian Equilibrium, each CRA follows a monotone rating strategy. Given a noisy signal s_i received by CRA_{*i*}, the optimal rating strategy is*

$$\hat{a}(s_i) = \begin{cases} \hat{a}(s_i), & \text{if } \hat{a}(s_i) > E(X|s_i); \\ E(X|s_i), & \text{otherwise.} \end{cases}$$

where

$$\hat{a}(s_i) = \chi \cdot (F(s_i))^{n-1},$$

This optimal rating strategy is a strictly increasing function of the noisy signal s_i .

The proof for this proposition is included in Appendix A.4. A CRA's equilibrium rating strategy in the unsolicited-rating system is again a piecewise function. When the condition $\chi \cdot (F(s_i))^{n-1} > E(X|s_i)$ is satisfied, the CRA_{*i*}'s optimal reporting rule is $\hat{a}(s_i)$. Otherwise the CRA_{*i*} follows a *truthful report* strategy and provides an indicative rating equal to the conditional expected asset return $E(X|s_i)$.

The *non-truthful report* function $\hat{a}(s_i)$ depends on the value of the issuer's publication fee χ , and the probability of CRA_{*i*}'s current signal being the highest among all. Again, we define **rating inflation** of CRA_{*i*} as the distance between the indicative rating and expected value of X : $\hat{a}(s_i) - E(X|s_i)$. As shown in the following corollary, the publication fees and competition drive the rating inflation in an opposite directions.

Corollary 4. *Under the unsolicited rating system, fix the number of CRAs, n , the rating inflation increases as the issuer's publication fee χ increases. In contrast, fix the issuer's publication fee χ , rating inflation declines as n increases.*

The corollary is derived from the comparative statics of the rating inflation with respect to the parameters χ and n . In the unsolicited rating system, a CRA responds positively to the issuer's publication fee in a similar manner as in the solicited rating system. As the issuer's publication fee χ increases, the gap between $\hat{a}(s_i)$ and $E(X|s_i)$ expands. Meanwhile, there is a rating inflation for a wider range of noisy signal s_i . Note that χ enters the rating function $\hat{a}(s_i)$ as a multiplication in the unsolicited rating system. It implies that the same amount of increment in publication fee χ may induce greater rating inflation compared to the degree of inflation in a solicited rating system. Figures A.1, A.2, and A.3 in the Appendix depict different patterns of rating inflation with regards to the value of χ , including the *truthful report* rating, fully inflated ratings, and partially inflated ratings.

The second part of the corollary, however, indicates that competition no longer encourages inflated rating in a credit market with unsolicited ratings. This is in a sharp contrast to the solicited rating system. Figure A.4 in the Appendix depicts the comparative static result with respect to n for a fixed χ value. As n increases, the function $\hat{a}(s_i)$ shifts down. In Appendix A.4, the comparative statics of the rating inflation with respect to n shows that $\partial(\hat{a}(s_i) - E(X|s_i))/\partial n < 0$. This implies that the degree of rating inflation decreases as competition increases. In an unsolicited rating system, all indicative ratings would eventually be published. Producing and publishing an inflated rating incurs a reputation loss with probability 1. On the other hand, as the number of CRAs grows, the probability for a CRA to be selected by the issuer decreases, which in turn reduces the expected payoff from earning the publication fee χ . Therefore, competition helps to mitigate CRAs' incentives

to inflate ratings in the unsolicited-rating system⁴.

4.2 Solicited, Unsolicited and Public-Pay: An Utility Comparison

In this subsection we compare investors' utility across the three market designs. Given a market size greater than one, investors' utility is higher under the unsolicited rating system than it is under the solicited rating system because the unsolicited ratings bypass the issuer's selection. In a credit market where a public-established third-party pays for the ratings, conflict of interest diminishes and all CRAs adopt the *truthful report* rating strategy. Under this public-pay system, investors' utility is the highest among all three credit market designs.

We first derive the investors' utility level under an unsolicited rating system. In the symmetric Perfect Bayesian Equilibrium, all the CRAs voluntarily publish their indicative ratings irrespective of the issuer's selection. Investors observe n credit ratings $\hat{\alpha}(s_1), \dots, \hat{\alpha}(s_n)$. Denote \hat{V}_n the investors' expected utility when observing n unsolicited ratings

$$\hat{V}_n = E_{X|S(1), \dots, S(n)}(X|\hat{\alpha}(s_1), \dots, \hat{\alpha}(s_n)) - r \cdot Var_{X|S(1), \dots, S(n)}(X|\hat{\alpha}(s_1), \dots, \hat{\alpha}(s_n))$$

We compare \hat{V}_n to V_n , the investors' expected utility under the solicited rating system, for a given number of CRAs n , and the issuer's publication fee payment χ .

Proposition 5. *When compared to the solicited-rating system, investors achieve higher expected utility in an unsolicited-rating system: $\hat{V}_n > V_n$.*

The proof of the proposition is included in Appendix A.6. Investors are better off under an unsolicited rating system. When n CRAs provide ratings in the market, investors observe n ratings under an unsolicited rating system, whereas they only observe one rating

⁴It is also worth noting that, when compared to the solicited-rating system, allowing unsolicited ratings may reduce, but not fully eliminate rating inflation. Specifically, in an unsolicited-rating system the rating inflation is less severe than that in a solicited-rating system when the following condition is satisfied:

$$\frac{\mu_X \cdot \sigma^2 + s_i \cdot \sigma_X^2}{\sigma^2 + \sigma_X^2} + \chi \cdot (1 - (F(s_i))^{n-1}) - \frac{\sigma_X^2}{\sigma^2 + \sigma_X^2} \cdot \frac{\int_{-\infty}^{s_i} [F(t)]^{n-1} dt}{[F(s_i)]^{n-1}} \geq 0$$

This condition is derived from the comparison of CRAs' optimal rating strategies between the solicited and unsolicited rating systems. Allowing unsolicited ratings can partially mitigate the degree of inflated ratings. Nonetheless, the inflation cannot be fully eliminated as long as the issuer adjusts the payment scheme to incentivize the CRAs to produce *non-truthful report* ratings.

under a solicited rating system. More rating observations reduce the conditional variance of the asset return X ; therefore, the investors' decision risks decline as their utility increases.

In the remaining subsection we consider a public-pay credit-rating model where the CRAs receive payment from a third party. This model retains the CRAs' role as market intermediaries and removes the conflict of interest inherited from the issuer-pay model. Specifically, a third party pays a fixed amount ξ to each CRA, which produces a rating and is willing to publish it. Each CRA commits to publishing the rating upon receiving the third party's payment ξ . A typical CRA $_i$'s optimization problem now becomes:

$$\xi - c - E_{X|s_i}|a_i - x|$$

Note that there is no uncertainty about the payment ξ , hence each CRA's profit maximization problem reduces to minimize its expected reputation loss $E_{X|s_i}|a_i - x|$. In this case, the optimal rating strategy for a typical CRA $_i$ is the *truthful report* strategy

$$E(X|s_i) = \frac{\mu_X \cdot \sigma^2 + s_i \cdot \sigma_X^2}{\sigma^2 + \sigma_X^2}.$$

This implies that there is no rating inflation under the public-pay system which removes the conflict of interest between CRAs and the issuer. Moreover, the above expression is not a function of n , the number of CRAs. This indicates that the degree of competition in the credit market has no impact on the CRAs' *truthful report* rating strategy. This result also indicates that, in contrast to an unsolicited rating system, increasing the number of CRAs does not result in inflated ratings and worsened decision quality. This is because the public-pay model removes the conflict of interest between the issuer and the CRAs effectively.

As revealed by Corollary 1, a CRA adopts the *truthful reporting* strategy under the solicited system when it is the monopolist CRA in the market. Next, we compare the social welfare of the public-pay system with n CRAs with that of the solicited rating system with a monopolist CRA. We denote n credit ratings $E(X|s_1), \dots, E(X|s_i), \dots, E(X|s_n)$ as the ratings of all participating CRAs under the public-pay system, and \bar{V}_n as the investors' expected utility conditional on these n observations:

$$\bar{V}_n = E_{X|S_{(1)}, \dots, S_{(n)}}(X|E(X|s_1), \dots, E(X|s_n)) - r \cdot Var_{X|S_{(1)}, \dots, S_{(n)}}(X|E(X|s_1), \dots, E(X|s_n))$$

We have the following result:

Proposition 6. *Compared to the solicited-rating system with a monopolistic CRA, in-*

investors achieve a higher expected utility level in the public-pay model: $\bar{V}_n > V_1$.

The proof of this proposition is included in the Appendix A.6. In both the public-pay system with n CRAs and the solicited rating system with a monopolist CRA, a participating CRA *truthfully reports* its estimate of the asset value. Yet the impacts on investors' utilities are unequal. Investors achieve a higher expected utility level in the public-pay system because investors observe rating from all n CRAs in the latter environment. The conditional variance of the asset declines as the number of observed ratings increases, which in turn reduces the investors' decision risk. In addition, combining the result from Proposition 2, we have $\bar{V}_n > V_1 > V_n$. This indicates that investors' utility from the public-pay system also dominates that from the solicited system with n rating agencies.

5 Conclusion

This paper investigates how competition among information intermediaries affects the credit rating in an issuer-pay credit-rating market. In our baseline game theory model, a bond issuer simultaneously pays low information fees to multiple CRAs to solicit the indicative ratings and always selects the highest rating to publish with an additional publication fee. We also consider two variations of the market design. In the unsolicited rating system, all CRAs are allowed to publish ratings without the issuer's selection and publication payment, which removes the issuer's selection effect. In the public-pay business model, a third party pays the CRAs to provide ratings, which effectively removes the conflict of interest.

We find that a monopolist CRA always truthfully reports its rating. If more than one CRA provides the ratings, a CRA may produce an inflated rating even in the presence of reputation cost if the payment comes from the issuer. As the number of CRAs increases, rating inflation increases in the solicited rating system but decreases in the unsolicited system. Reputation disciplines the CRAs more effectively in the unsolicited system because every CRA is exposed to reputation cost if it reveals an unsolicited rating. If a CRA receives the payment without uncertainty from a third party, it will truthfully report its estimate of the asset value.

Investors strictly prefer to have at least one CRA in the market, because the CRA has an information advantage over investors. Under the solicited rating system, the competition among multiple CRAs undermines investors' utility when compared to the monopolistic CRA case. Given a market size greater than one, the unsolicited ratings increase investors'

utility because investors observe more ratings, leading to a lower decision risk. The public-pay model results in the highest utility among the three market design. When we evaluate social welfare, taking into account the CRAs' information cost and reputation cost, as well as the investors' utility, we find that it might not be socially optimal to acquire information through the CRAs unless the information cost is low.

Our findings echo the empirical evidence that suggests that competition among rating agencies deteriorates the quality of ratings under the issuer-pay model. Reputation is not enough to keep the CRAs from inflating rating because whoever provides the highest rating will be selected by the issuer to publish the rating and receive additional compensation. The unsolicited rating removes the selection effect, but conflict of interest remains. According to the theoretical model, the public-pay system appears to be the most desirable market design for investors because conflict of interest vanishes. In the reality, the Risk Management Institute (RMI) at the National University of Singapore has been the pioneer to provide credit ratings as a public good for 25,000 companies in 14 economies after the Great Recession.

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A Appendix

A.1 Proof of Proposition 1

We first derive the distribution of $X|S_i$, i.e. the random variable X conditional on its noisy realization $S_i = s_i$. The result is used in deriving Proposition 1: upon getting a noisy signal s_i , CRA_i adopts Bayes' Rule to calculate the bond's expected return and the variance of the return.

$$\begin{aligned}
E(X|s_i) &= \int_{-\infty}^{\infty} x f(x|s_i) dx \\
&= \int_{-\infty}^{\infty} x \frac{f(x) f(s_i|x)}{f(s_i)} dx \\
&= \int_{-\infty}^{\infty} x \frac{\frac{1}{\sigma_x \sqrt{2\pi}} \exp\left(-\frac{(x-\mu_X)^2}{2\sigma_X^2}\right) \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(s-x)^2}{2\sigma^2}\right)}{\frac{1}{\sqrt{\sigma_X^2 + \sigma^2} \sqrt{2\pi}} \exp\left(-\frac{(s-\mu_X)^2}{2(\sigma_X^2 + \sigma^2)}\right)} dx \\
&= \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi} \frac{\sigma_x \sigma}{\sqrt{\sigma_X^2 + \sigma^2}}} \exp\left(-\frac{\left(x - \frac{\sigma^2 \mu_X + \sigma_X^2 s_i}{\sigma_X^2 + \sigma^2}\right)^2}{2\sigma_X^2 \sigma^2 \frac{1}{\sigma_X^2 + \sigma^2}}\right) dx \\
&= \frac{\mu_X \sigma^2 + s_i \sigma_X^2}{\sigma^2 + \sigma_X^2} \\
Var(X|s_i) &= \frac{\sigma_X^2 \sigma^2}{\sigma_X^2 + \sigma^2}
\end{aligned}$$

where f is the density function of normal distribution. Since $X \sim N(\mu_X, \sigma_X^2)$, $\epsilon \sim N(0, \sigma^2)$, $s = X + \epsilon$, and $corr(\epsilon_i, X) = 0$, we have $s \sim N(\mu_X, \sigma_X^2 + \sigma^2)$. Therefore, we have $X|s_i \sim N\left(\frac{\sigma^2 \mu_X + \sigma_X^2 s_i}{\sigma_X^2 + \sigma^2}, \frac{\sigma_X^2 \sigma^2}{\sigma_X^2 + \sigma^2}\right)$ from the above derivation.

Denote each CRA's expected payoff as:

$$\begin{aligned}
E\pi &= [\lambda + \chi - c - |a_i - x|] \cdot Prob(win) + [\lambda - c] \cdot [1 - Prob(win)] \\
&= [\chi - |a_i - x|] \cdot Prob(win) + [\lambda - c]
\end{aligned}$$

Since CRA_i does not know the exact value of x and forms expectation based on the observed signal s_i , we replace x with $E(X|s_i)$. In addition, given the symmetry of $a(s_i)$, $Prob(win) = Prob(a_i \geq a_j, i \neq j) = [F(a^{-1}(a_i))]^{m-1}$. Hence, CRA_i 's optimization problem can be written as:

$$\max_{a_i} [\chi - |a_i - E(X|s_i)|] \cdot Prob(a_i \geq a_j, i \neq j) + [\lambda - c]$$

Case 1: $a_i \leq E(X|s_i)$, CRA_i 's expected payoff is:

$$E\pi = [\chi - E(X|s_i) + a_i] \cdot Prob(a_i \geq a_j, i \neq j) + [\lambda - c],$$

which is an increasing function of a_i . Notice that $Prob(a_i \geq a_j, i \neq j)$ also increases as a_i gets larger. Therefore, we have a corner solution for the optimization problem and the maximum is attained at:

$$E(X|s_i) = \frac{\mu_X \sigma^2 + s_i \sigma_X^2}{\sigma^2 + \sigma_X^2}$$

Case 2: $a_i > E(X|s_i)$, CRA_i 's expected payoff is:

$$\begin{aligned} E\pi &= [\chi + E(X|s_i) - a_i] \cdot [F(a^{-1}(a_i))]^{m-1} + [\lambda - c] \\ &= \left[\chi + \frac{\mu_X \sigma^2 + s_i \sigma_X^2}{\sigma^2 + \sigma_X^2} - a_i \right] \cdot [F(a^{-1}(a_i))]^{m-1} + [\lambda - c] \end{aligned}$$

As a_i increases, $\chi + \frac{\mu_X \sigma^2 + s_i \sigma_X^2}{\sigma^2 + \sigma_X^2} - a_i$ decreases while $[F(a^{-1}(a_i))]^{m-1}$ increases. Each CRA in this case faces a tradeoff between reputation loss and enhanced probability of getting the publication fee. Therefore, the optimal reporting rule $a(s_i)$ is an interior solution of the above problem. We derive the first order condition w.r.t. a_i :

$$\left[\chi + \frac{\mu_X \sigma^2 + s_i \sigma_X^2}{\sigma^2 + \sigma_X^2} - a_i \right] (m-1) [F(a^{-1}(a_i))]^{m-2} \cdot f(a^{-1}(a_i)) \cdot \frac{1}{a'(a^{-1}(a_i))} - [F(a^{-1}(a_i))]^{m-1} = 0$$

We replace a_i with $a(s_i)$, replace $a^{-1}(a_i)$ with s_i , and rearrange terms:

$$\left[\chi + \frac{\mu_X \sigma^2 + s_i \sigma_X^2}{\sigma^2 + \sigma_X^2} \right] (m-1) [F(s_i)]^{m-2} f(s_i) = [F(s_i)]^{m-1} a'(s_i) + a(s_i) (m-1) [F(s_i)]^{m-2} f(s_i)$$

Integrate both sides from $-\infty$ to s_i and rearrange terms, we have:

$$a(s_i) = \frac{\mu_X \sigma^2 + s_i \sigma_X^2}{\sigma^2 + \sigma_X^2} + \chi - \frac{\sigma_X^2}{\sigma^2 + \sigma_X^2} \cdot \frac{\int_{-\infty}^{s_i} [F(t)]^{m-1} dt}{[F(s_i)]^{m-1}}$$

The reporting rule is the expected value of true return conditional on signal s_i , $\frac{\mu_X \sigma^2 + s_i \sigma_X^2}{\sigma^2 + \sigma_X^2}$, plus the publication fee, χ , minus the adjustment term as in an auction setting.

To sum up, when $\chi - \frac{\sigma_X^2}{\sigma^2 + \sigma_X^2} \cdot \frac{\int_{-\infty}^{s_i} [F(t)]^{m-1} dt}{[F(s_i)]^{m-1}} > 0$, CRA_i 's optimal reporting rule is $a(s_i)$; otherwise it is the true expectation of X given the noisy signal s_i : $E(X|s_i)$.

It remains to show that the improper integral $\int_{-\infty}^{s_i} [F(s)]^{m-1} ds$ exists. First notice that $\int_{-\infty}^{s_i} [F(s)]^{m-1} ds \leq \int_{-\infty}^{s_i} F(s) ds$. Therefore it is sufficient to prove that the latter is bounded

above. Since $s \sim N(\mu_X, \sigma_X^2 + \sigma^2)$, $f(s) = \frac{\exp(-\frac{(s-\mu_X)^2}{2(\sigma_X^2 + \sigma^2)})}{\sqrt{2\pi(\sigma_X^2 + \sigma^2)}}$. Let $k^2 = 2(\sigma_X^2 + \sigma^2)$, we have the following:

$$\begin{aligned}
\int_{-\infty}^{s_i} F(s)ds &= \int_{-\infty}^{s_i} \int_{-\infty}^s f(t)dt ds \\
&= \int_{-\infty}^{s_i} \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\frac{s-\mu_X}{k}} \exp(-t^2) dt ds \\
&= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\frac{s_i-\mu_X}{k}} \int_{kt+\mu_X}^{s_i} \exp(-t^2) ds dt \\
&= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\frac{s_i-\mu_X}{k}} \exp(-t^2)(s_i - kt - \mu_X) dt
\end{aligned}$$

Change variable $s = kt + \mu_X$, the above equals:

$$\begin{aligned}
&\frac{1}{\sqrt{\pi}k}(s_i - \mu_X) \int_{-\infty}^{s_i} \exp(-\frac{(s - \mu_X)^2}{k^2}) ds - \frac{k}{\sqrt{\pi}}(-\frac{1}{2}) \exp(-t^2) \Big|_{-\infty}^{\frac{s_i-\mu_X}{k}} \\
&= \frac{1}{\sqrt{2\pi\sigma_X^2 + \sigma^2}}(s_i - \mu_X) \int_{-\infty}^{s_i} \exp(-\frac{(s - \mu_X)^2}{2(\sigma_X^2 + \sigma^2)}) ds + \frac{\sqrt{2(\sigma_X^2 + \sigma^2)}}{2\sqrt{\pi}} \exp(-(\frac{s_i - \mu_X}{k})^2 - 0) \\
&= (s_i - \mu_X)F(s_i) + (\sigma_X^2 + \sigma^2)f(s_i) \\
&\leq (\mu_X + 3\sqrt{\sigma_X^2 + \sigma^2} - \mu_X) \times 1 + (\sigma_X^2 + \sigma^2)f(\mu_X) \\
&= 3\sqrt{\sigma_X^2 + \sigma^2} + \frac{\sqrt{\sigma_X^2 + \sigma^2}}{\sqrt{2\pi}}
\end{aligned}$$

Note that we focus on $s_i \in [\mu_X - 3\sqrt{\sigma_X^2 + \sigma^2}, \mu_X + 3\sqrt{\sigma_X^2 + \sigma^2}]$. The improper integral exists, thus the above-specified CRAs' optimal rating function $a(s_i)$ is well-defined.

Next we show that the reporting rule is monotone increasing in s_i . Rewriting $a(s_i)$ as:

$$a(s_i) = \frac{\sigma^2}{\sigma_X^2 + \sigma^2} \mu_X + \chi + \frac{\sigma_X^2}{\sigma_X^2 + \sigma^2} \left(s_i - \frac{\int_{-\infty}^{s_i} [F(t)]^{m-1} dt}{[F(s_i)]^{m-1}} \right)$$

It suffices to prove that

$$d\left(s_i - \frac{\int_{-\infty}^{s_i} [F(t)]^{m-1} dt}{[F(s_i)]^{m-1}}\right) / ds_i > 0.$$

Let $G(s_i) := \int_{-\infty}^{s_i} [F(t)]^{m-1} dt$. Then $G'(s_i) = [F(s_i)]^{m-1}$ and $G''(s_i) = (m-1)[F(s_i)]^{m-2} f(s_i) >$

0, where f and F are the *pdf* and *cdf* of Normal Distribution. Then we have:

$$\begin{aligned} \left[\frac{G(s_i)}{G'(s_i)} \right]' &= \frac{G'(s_i)G'(s_i) - G(s_i)G''(s_i)}{G'(s_i)G'(s_i)} \\ &= 1 - \frac{G(s_i)G''(s_i)}{[G'(s_i)]^2} \end{aligned}$$

Since $\frac{G(s_i)G''(s_i)}{[G'(s_i)]^2} \in (0, 1)$, we have $\left[\frac{G(s_i)}{G'(s_i)} \right]' \in (0, 1)$. The symmetric and monotone reporting strategies exist. **Q.E.D**

A.2 Proof of Lemma 1

The first-order derivative of $a(s_i)$ with respect to s_i is

$$a'(s_i) = \frac{\sigma_X^2}{\sigma^2 + \sigma_X^2} \cdot \frac{(m-1)f(s_i) \int_{-\infty}^{s_i} [F(t)]^{m-1} dt}{[F(s_i)]^m}$$

Define $B(s_i) \doteq s_i - \frac{\int_{-\infty}^{s_i} (F(t))^{m-1} dt}{(F(s_i))^{m-1}}$. Notice that showing $a'(s_i) \leq \frac{dE(X|s_i)}{ds_i}$ is equivalent to demonstrating $B'(s_i) < 1$.

Claim: $B'(s_i) = \frac{(m-1)f(s_i) \int_{-\infty}^{s_i} [F(t)]^{m-1} dt}{[F(s_i)]^m} \leq 1$. Proof: First we will show that $\lim_{s \rightarrow -\infty} B'(s_i) = 1$. Use L'Hopital Rule repeatedly we will get:

$$\begin{aligned} \lim_{s \rightarrow -\infty} B'(s_i) &= \lim_{s \rightarrow -\infty} \frac{(m-1)f(s_i) \int_{-\infty}^{s_i} [F(t)]^{m-1} dt}{[F(s_i)]^m} \\ &= \lim_{s \rightarrow -\infty} \frac{(m-1)[f'(s_i) \int_{-\infty}^{s_i} [F(t)]^{m-1} dt + f(s_i)F(s_i)^{m-1}]}{(F(s_i))^{m-1} f(s_i)} \\ &= -\frac{1}{\sigma_X^2 + \sigma^2} \lim_{s \rightarrow -\infty} \frac{(s_i - \mu_X) \int_{-\infty}^{s_i} [F(t)]^{m-1} dt}{(F(s_i))^{m-1}} \\ &= 0 + 1 = 1 \end{aligned}$$

Now we need to show $B'(s_i) < 1$. Suppose $\exists s_0$ s.t. $B'(s_0) \geq 1$. Then $\exists s_{00}$ s.t. $B'(s_{00}) \geq 1$ and $B''(s_{00}) = 0$ (the maximum of $B'(s_i)$). However, it's easy to show that $B''(s_i) = \frac{(m-1)[F(s_i)]^{m-1} \int_{-\infty}^{s_i} [F(t)]^{m-1} dt [f'(s_i)F(s_i) - nf^2(s_i)] + (m-1)f(s_i)[F(s_i)]^{m-1}}{[F(s_i)]^{2m-1}} < 0$. A contradiction. Therefore $B'(s_i) < 1$ holds for all finite s_i 's.

As $s_i \rightarrow +\infty$, $F(s_i) \rightarrow 1$ and $f(s_i) \rightarrow 0$, thus $a'(s_i) \rightarrow 0$. Hence, when $s_i \rightarrow [\mu_X - 3\sqrt{\sigma_X^2 + \sigma^2}, \mu_X + 3\sqrt{\sigma_X^2 + \sigma^2}]^-$, $a'(s_i) < \frac{\sigma_X^2}{\sigma^2 + \sigma_X^2} = \frac{dE(X|s_i)}{ds_i}$. Moreover, the second-order

derivative shows:

$$a''(s_i) = \frac{\sigma_X^2}{\sigma^2 + \sigma_X^2} \cdot \frac{(m-1)[F(s_i)]^{m-1} \int_{-\infty}^{s_i} [F(t)]^{m-1} dt [f'(s_i)F(s_i) - n f^2(s_i)] + (m-1)f(s_i)[F(s_i)]^{m-1}}{[F(s_i)]^{2m-1}}$$

The above is always negative for any $n \geq 2$. Therefore the function $a(s_i)$ is concave within the range $[\mu_X - 3\sqrt{\sigma_X^2 + \sigma^2}, \mu_X + 3\sqrt{\sigma_X^2 + \sigma^2}]$. **Q.E.D**

A.3 Proof for Proposition 2

We first derive the distribution of the highest signal $S_{(n)}$. Denote $S_{(n)} = s_{(n)}$ as the *n*th order statistic among all independently drawn signal observations $\{s_{(1)}, \dots, s_{(n)}\}$, namely, $s_{(n)} = \max\{s_1, \dots, s_n\}$ where each s_i is a random draw from the CDF $S_i \sim F(\cdot)$ of a normal distribution with mean μ_X and variance $\sigma_X^2 + \sigma_i^2$. The distribution of $S_{(n)}$ can be written as:

$$\hat{F}_{S_{(n)}}(s_{(n)}) = Pr(S_{(n)} \leq s_{(n)}) = Pr(S_1 \leq s_{(n)}, \dots, S_n \leq s_{(n)}) = (F_{S_i}(s_{(n)}))^n$$

and the corresponding pdf $f(\cdot)$ is

$$\hat{f}_{S_{(n)}}(s_{(n)}) = n \cdot (F_{S_i}(s_{(n)}))^{n-1} \cdot f_{S_i}(s_{(n)})$$

Therefore, the unconditional density of $S_{(n)} = s$ is

$$f(S_{(n)} = s) = n \cdot \left(\int_{-\infty}^s \frac{1}{\sqrt{\sigma_X^2 + \sigma^2} \cdot \sqrt{2\pi}} \cdot \exp\left(-\frac{(t - \mu_X)^2}{2(\sigma_X^2 + \sigma^2)}\right) dt \right)^{n-1} \cdot \frac{1}{\sqrt{\sigma_X^2 + \sigma^2} \cdot \sqrt{2\pi}} \cdot \exp\left(-\frac{(s - \mu_X)^2}{2(\sigma_X^2 + \sigma^2)}\right)$$

And the conditional density is

$$\begin{aligned} f_{S_{(n)}|X}(S_{(n)} = s|X = x) &= n \cdot (F_{S_i|X}(S_i = s|X = x))^{n-1} \cdot f_{S_i|X}(S_i = s|X = x) \\ &= n \cdot \left(\int_{-\infty}^s \frac{1}{\sqrt{2\pi \cdot \sigma^2}} \cdot \exp\left(-\frac{(t - x)^2}{2\sigma^2}\right) dt \right)^{n-1} \cdot \frac{1}{\sqrt{2\pi \cdot \sigma^2}} \cdot \exp\left(-\frac{(s - x)^2}{2\sigma^2}\right) \end{aligned}$$

And the distribution of X conditional on the highest signal $S_{(n)} = s$ is selected is

$$\begin{aligned}
f_{S_{(n)}|X}(X = x|S_{(n)} = s) &= \frac{f_X(x) \cdot f_{S_{(n)}|X}(S_{(n)} = s|X = x)}{f(S_{(n)} = s)} \\
&= \frac{1}{\sqrt{2\pi \cdot \sigma_x^2}} \exp\left(-\frac{(x - \mu_X)^2}{2\sigma_X^2}\right) \cdot \frac{f_{S_{(n)}|X}(S_{(n)} = s|X = x)}{f(S_{(n)} = s)} \\
&= \frac{1}{\sqrt{2\pi \cdot \sigma_x^2}} \cdot \left(\sqrt{\frac{\sigma_x^2 + \sigma^2}{\sigma^2}}\right)^n \cdot \left(\frac{\int_{-\infty}^s \exp\left(-\frac{(t-x)^2}{2\sigma^2}\right) dt}{\int_{-\infty}^s \exp\left(-\frac{(t-\mu_X)^2}{2(\sigma_X^2 + \sigma^2)}\right) dt}\right)^{n-1} \\
&\quad \cdot \exp\left(\frac{-[\sigma^2 \cdot (x - \mu_X) - \sigma_X^2 \cdot (s - x)]^2}{2 \cdot \sigma_X^2 \cdot \sigma^2 \cdot (\sigma_X^2 + \sigma^2)}\right)
\end{aligned}$$

The expectation and variance can be derived accordingly:

$$\begin{aligned}
E_{S_{(n)}|X}(X = x|S_{(n)} = s) &= \int_{-\infty}^{+\infty} x \cdot f_{S_{(n)}|X}(X = x|S_{(n)} = s) dx \\
Var_{S_{(n)}|X}(X = x|S_{(n)} = s) &= \int_{-\infty}^{+\infty} x^2 \cdot f_{S_{(n)}|X}(X = x|S_{(n)} = s) dx - (E_{S_{(n)}|X}(X = x|S_{(n)} = s))^2
\end{aligned}$$

Next, we consider the investors' utilities in terms of the expectation and variance of the conditional asset return. Denote V_0 the investors' expected utility when no credit rating is published by the issuer. Thus we have:

$$V_0 = E_{X|S_0}(X|s_0) - r \cdot Var_{X|S_0}(X|s_0)$$

And denote V_1 the investors' expected utility when there is one monopolistic CRA which observes a noisy signal $S_i = s_i$ and produces a credit rating:

$$V_1 = E_{X|S_i}(X|E(X|s_i)) - r \cdot Var_{X|S_i}(X|E(X|s_i))$$

By the assumption $\sigma_0^2 > \sigma_i^2, \forall i$, we have $Var_{X|S_0}(X|s_0) > Var_{X|S_i}(X|E(X|s_i))$. Since we look into the interval $s_0, s_i \in [\mu_X - 3\sqrt{\sigma_X^2 + \sigma^2}, \mu_X + 3\sqrt{\sigma_X^2 + \sigma^2}], \forall i$, there exists a threshold value \bar{r}_0 such that when $r \geq \bar{r}_0$, we have

$$E_{X|S_i}(X|E(X|s_i)) - r \cdot Var_{X|S_i}(X|E(X|s_i)) > E_{X|S_0}(X|s_0) - r \cdot Var_{X|S_0}(X|s_0)$$

Denote V_n the investors' expected utility when the issuer selects and publishes one out of n indicative ratings:

$$\begin{aligned}
V_n &= E_{X|S(n)}(X|\alpha(s(n))) - r \cdot \text{Var}_{X|S(n)}(X|\alpha(s(n))) \\
&= \begin{cases} E_{X|S(n)}(X|\tilde{a}(s(n))) - r \cdot \text{Var}_{X|S(n)}(X|\tilde{a}(s(n))) & \text{if } \tilde{a}(s_i) > E(X|s_i); \\ E_{X|S(n)}(X|E(X|s(n))) - r \cdot \text{Var}_{X|S(n)}(X|E(X|s(n))), & \text{if } \tilde{a}(s_i) \leq E(X|s_i) \end{cases}
\end{aligned}$$

First consider the case in which $\tilde{a}(s_i) \leq E(X|s_i)$. Observe that $\text{Var}_{X|S(n)}(X|E(X|s(n))) > \text{Var}_{X|S_i}(X|E(X|s_i))$. We focus on the values of $s_i \in [\mu_X - 3\sqrt{\sigma_X^2 + \sigma^2}, \mu_X + 3\sqrt{\sigma_X^2 + \sigma^2}]$, $\forall i$. Hence there exists threshold value \bar{r}_1 such that when $r \geq \bar{r}_1$, we have

$$E_{X|S(n)}(X|E(X|s(n))) - r \cdot \text{Var}_{X|S(n)}(X|E(X|s(n))) < E_{X|S_i}(X|E(X|s_i)) - r \cdot \text{Var}_{X|S_i}(X|E(X|s_i))$$

Then consider the case in which $\tilde{a}(s_i) > E(X|s_i)$. We have $\text{Var}_{X|S(n)}(X|\tilde{a}(s(n))) > \text{Var}_{X|S(n)}(X|E(X|s(n)))$. Therefore, for $r \geq \bar{r}_1$, the above inequality implies

$$E_{X|S(n)}(X|\tilde{a}(s(n))) - r \cdot \text{Var}_{X|S(n)}(X|\tilde{a}(s(n))) < E_{X|S_i}(X|E(X|s_i)) - r \cdot \text{Var}_{X|S_i}(X|E(X|s_i))$$

Thus we show that for any $n > 1$, the investors' utility $V_n < V_1$. **Q.E.D**

A.4 Proof of Proposition 4

In the case with unsolicited ratings, each CRA's expected payoff is:

$$\begin{aligned}
E\pi &= [\lambda + \chi - c - |a_i - x|] \cdot \text{Prob}(\text{win}) + [\lambda - c - |a_i - x|] \cdot [1 - \text{Prob}(\text{win})] \\
&= \chi \cdot \text{Prob}(\text{win}) + [\lambda - c] - |a_i - x|
\end{aligned}$$

Since CRA_i does not know the exact value of x and forms expectation based on the observed signal s_i , we replace x with $E(X|s_i)$. In addition, given the symmetry of $a(s_i)$, $\text{Prob}(\text{win}) = \text{Prob}(a_i \geq a_j, i \neq j) = [F(a^{-1}(a_i))]^{m-1}$. Hence, CRA_i 's optimization problem can be written as:

$$\max_{a_i} \chi \cdot \text{Prob}(a_i \geq a_j, i \neq j) + [\lambda - c] - |a_i - E(X|s_i)|$$

Case 1: $a_i \leq E(X|s_i)$, CRA_i 's expected payoff is:

$$E\pi = \chi \cdot \text{Prob}(a_i \geq a_j, i \neq j) + [\lambda - c] - E(X|s_i) + a_i$$

which is an increasing function of a_i . Therefore, we have a corner solution for the optimization problem and the maximum is attained at:

$$E(X|s_i) = \frac{\mu_X \sigma^2 + s_i \sigma_X^2}{\sigma^2 + \sigma_X^2}$$

Case 2: $a_i > E(X|s_i)$, CRA $_i$'s expected payoff is:

$$\begin{aligned} E\pi &= \chi \cdot [F(a^{-1}(a_i))]^{m-1} + [\lambda - c] + [E(X|s_i) - a_i] \\ &= \chi \cdot [F(a^{-1}(a_i))]^{m-1} + [\lambda - c] + \left[\frac{\mu_X \sigma^2 + s_i \sigma_X^2}{\sigma^2 + \sigma_X^2} - a_i \right] \end{aligned}$$

As a_i increases, $\chi + \frac{\mu_X \sigma^2 + s_i \sigma_X^2}{\sigma^2 + \sigma_X^2} - a_i$ decreases while $[F(a^{-1}(a_i))]^{m-1}$ increases. Each CRA in this case faces a tradeoff between loss in reputation and increment in probability of getting the publication fee. Therefore, the optimal reporting rule $a(s_i)$ is an interior solution of the above problem. We derive the first order condition w.r.t. a_i :

$$\chi(m-1)[F(a^{-1}(a_i))]^{m-2} \cdot f(a^{-1}(a_i)) \cdot \frac{1}{a'(a^{-1}(a_i))} - 1 = 0$$

We replace a_i with $a(s_i)$, replace $a^{-1}(a_i)$ with s_i , and rearrange terms:

$$\chi(m-1)[F(s_i)]^{m-2} f(s_i) = a'(s_i)$$

Integrate with respect to s_i , we have:

$$a(s_i) = \chi \cdot (F(s_i))^{m-1}$$

Next we will show that the reporting rule is monotone increasing in s_i . The first order derivative shows:

$$a'(s_i) = \chi(m-1)(F(s_i))^{m-2} f(s_i) > 0$$

Therefore the reporting function is strictly increasing and symmetric.

Now we show Corollary 4 by deriving the comparative statics of $\hat{a}(s_i)$ with respect to n . The partial derivative is $\partial(\hat{a}(s_i) - E(X|s_i))/\partial n = \chi \cdot (F(s_i))^{n-1} \cdot \ln(F(s_i)) < 0$, since $(F(s_i)) < 1$. **Q.E.D**

A.5 Graphic Illustration for Partial Inflation of Unsolicited Ratings

Figure A.1 and A.2 depict the two extreme cases as regards the degree of rating inflation: in the former case, CRA_i adopts the *truthful report* strategy within the range $s_i \in [\mu_X - 3\sqrt{\sigma_X^2 + \sigma^2}, \mu_X + 3\sqrt{\sigma_X^2 + \sigma^2}]$, whereas in the latter situation, CRA_i inflates its indicative rating for all possible values of signal s_i .

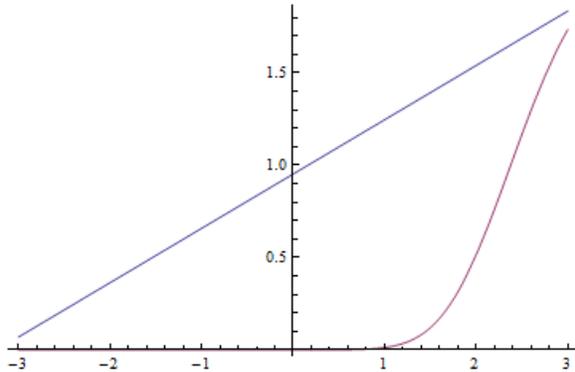


Figure A.1: Non-Inflated Report
Parameter values:

$\mu_X = 1.38, \sigma = 0.96, \sigma_X = 0.62.$
 $n = 6, \chi = 2.16.$ The blue line is depicted for $E(S|s_i)$ while the purple curve for $\hat{a}(s_i)$.

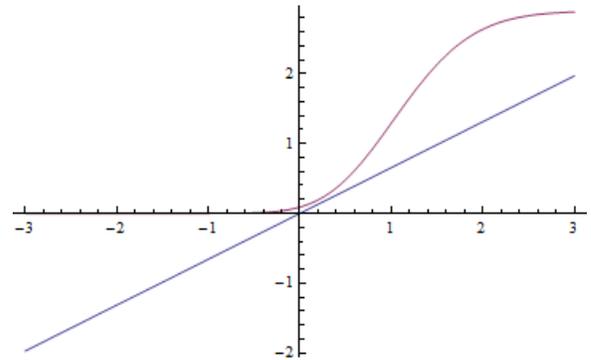


Figure A.2: Inflated Report
Parameter values:

$\mu_X = 0, \sigma = 0.96, \sigma_X = 1.33. n = 6, \chi = 2.9.$
The blue line is depicted for $E(X|s_i)$ while the purple curve for $\hat{a}(s_i)$.

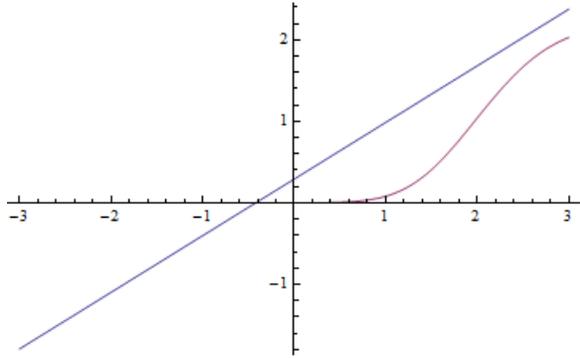
Figure A.3 depicts the four other cases with partially inflated indicative ratings in the unsolicited-rating system.

Figure A.4 depicts the comparative static result with respect to n for a fixed χ value.

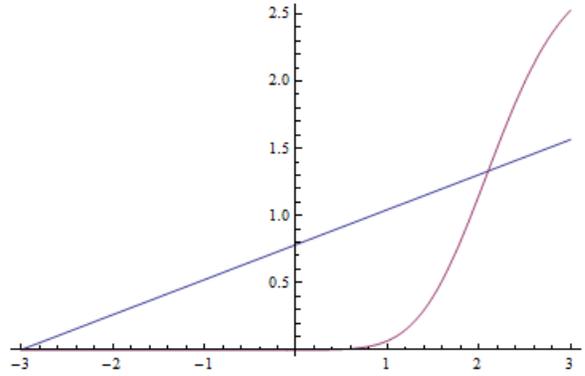
A.6 Proof of Proposition 5 and 6

We first derive the distribution of $X|S_1, \dots, S_n$, i.e. the random variable X conditional on the joint probability distribution of random variables S_1, \dots, S_n . Because of the noisy signals s_i 's are i.i.d. from the normal distribution $N(\mu_X, \sigma_X^2 + \sigma^2)$, the unconditional density of the joint distribution of signals is

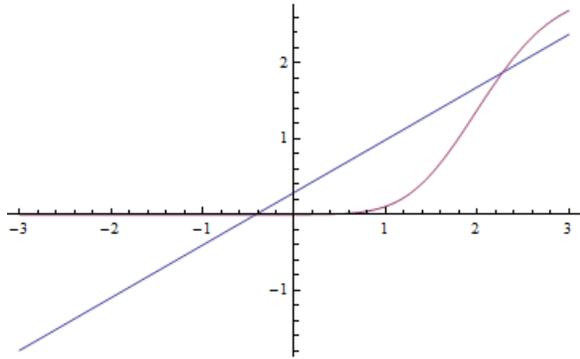
$$f_{S_1, \dots, S_n}(s_1, \dots, s_n) = \prod_{i=1}^n \frac{1}{\sqrt{\sigma_X^2 + \sigma^2} \sqrt{2\pi}} \cdot \exp\left(-\frac{(s_i - \mu_X)^2}{2(\sigma_X^2 + \sigma^2)}\right)$$



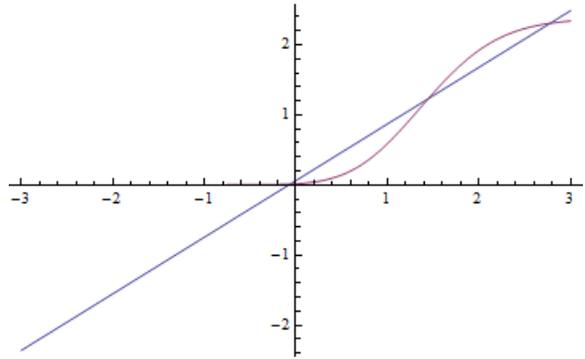
Parameter values: $\mu_X = 0.96, \sigma = 0.96, \sigma_X = 1.45. n = 6, \chi = 2.21$. The blue line is for $E(S|s_i)$ while the purple curve for $a(s_i)$.



Parameter values: $\mu_X = 0.96, \sigma = 0.96, \sigma_X = 0.50. n = 6, \chi = 2.85$. The blue line is for $E(S|s_i)$ while the purple curve for $a(s_i)$.



Parameter values: $\mu_X = 0.96, \sigma = 0.96, \sigma_X = 1.45. n = 6, \chi = 2.93$. The blue line is for $E(S|s_i)$ while the purple curve for $a(s_i)$.



Notes: Parameter values: $\mu_X = 0.33, \sigma = 0.96, \sigma_X = 1.97. n = 6, \chi = 2.37$. The blue line is for $E(X|s_i)$ while the purple curve for $a(s_i)$.

Figure A.3: 4 cases for partially inflated ratings.

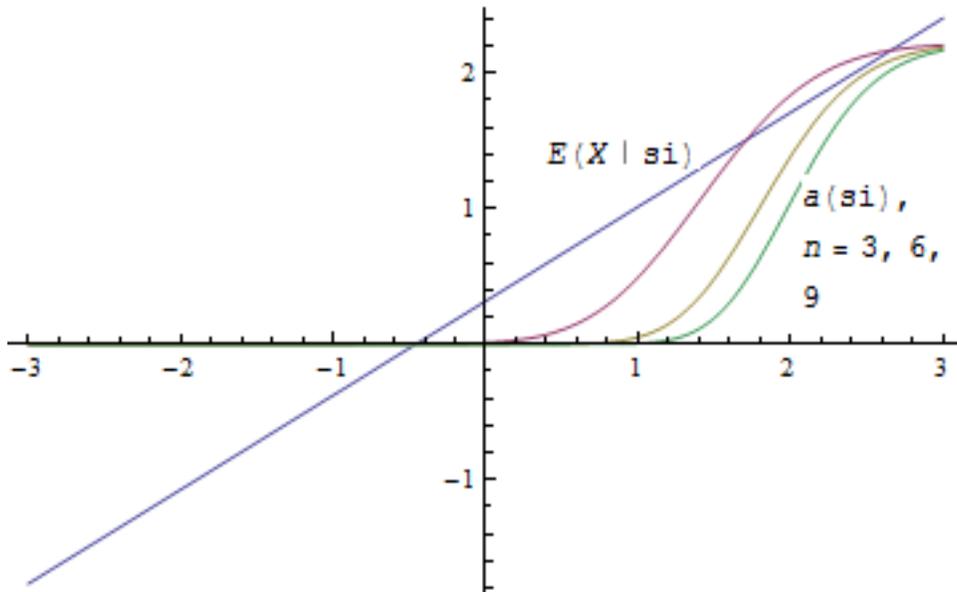


Figure A.4: Comparative Statics w.r.t m

Parameter values: $\mu_X = 1.05, \sigma = 0.39, \sigma_X = 0.59. \chi = 2.22$ fixed. The blue line is depicted for $E(X|s_i)$ while the purple, yellow, and green curve for $\hat{a}(s_i)$ when $n = 3, 6, 9$ respectively.

And the conditional joint density is

$$f_{S_i|X}(S_1 = s, \dots, S_n = s|X = x) = \prod_{i=1}^n \left(\frac{1}{\sqrt{2\pi \cdot \sigma^2}} \cdot \exp\left(-\frac{(s_i - x)^2}{2 \cdot \sigma^2}\right) \right)$$

And the distribution of X conditional on n noisy signals s_1, \dots, s_n is

$$\begin{aligned} f(X|S_1 = s_1, \dots, S_n = s_n) &= \frac{f_X(x) \cdot f_{S_i|X}(S_1 = s_1, \dots, S_n = s_n|X = x)}{f_{S_1, \dots, S_n}(s_1, \dots, s_n)} \\ &= \frac{1}{\sqrt{2\pi \cdot \sigma_x^2}} \exp\left(-\frac{(x - \mu_X)^2}{2 \cdot \sigma_x^2}\right) \cdot \frac{\prod_{i=1}^n \left(\frac{1}{\sqrt{2\pi \cdot \sigma^2}} \cdot \exp\left(-\frac{(s_i - x)^2}{2 \cdot \sigma^2}\right) \right)}{\prod_{i=1}^n \frac{1}{\sqrt{\sigma_x^2 + \sigma^2}} \cdot \exp\left(-\frac{(s_i - \mu_X)^2}{2(\sigma_x^2 + \sigma^2)}\right)} \end{aligned}$$

So the conditional expectation and variance can be derived accordingly:

$$\begin{aligned} E(X|S_1 = s_1, \dots, S_n = s_n) &= \int_{-\infty}^{+\infty} x \cdot f(X|S_1 = s_1, \dots, S_n = s_n) dx \\ Var(X|S_1 = s_1, \dots, S_n = s_n) &= \int_{-\infty}^{+\infty} x^2 \cdot f(X|S_1 = s_1, \dots, S_n = s_n) dx - (E(X|S_1 = s_1, \dots, S_n = s_n))^2 \end{aligned}$$

We first show Proposition 5 by comparing the investors' utility under the solicited rating with that under the unsolicited rating system. As before, V_n denotes the investors' expected utility when the issuer selects and publishes one out of n indicative ratings. And \hat{V}_n denotes the investors' utility under an unsolicited rating system

$$\begin{aligned} V_n &= \begin{cases} E_{X|S(n)}(X|\tilde{a}(s(n))) - r \cdot Var_{X|S(n)}(X|\tilde{a}(s(n))) & \text{if } \tilde{a}(s_i) > E(X|s_i); \\ E_{X|S(n)}(X|E(X|s(n))) - r \cdot Var_{X|S(n)}(X|E(X|s(n))), & \text{if } \tilde{a}(s_i) \leq E(X|s_i) \end{cases} \\ \hat{V}_n &= E_{X|S(1), \dots, S(n)}(X|\hat{\alpha}(s_1), \dots, \hat{\alpha}(s_n)) - r \cdot Var_{X|S(1), \dots, S(n)}(X|\hat{\alpha}(s_1), \dots, \hat{\alpha}(s_n)) \\ &= \begin{cases} E_{X|S(1), \dots, S(n)}(X|\hat{\alpha}(s_1), \dots, \hat{\alpha}(s_n)) - r \cdot Var_{X|S(1), \dots, S(n)}(X|\hat{\alpha}(s_1), \dots, \hat{\alpha}(s_n)), & \text{if } \hat{\alpha}(s_i) > \\ E_{X|S(1), \dots, S(n)}(X|E(X|s_1), \dots, E(X|s_n)) - r \cdot Var_{X|S(1), \dots, S(n)}(X|E(X|s_1), \dots, E(X|s_n)), & \text{o/w.} \end{cases} \end{aligned}$$

The conditional variance on n observations is smaller than that with only one observation:

$$Var_{X|S(1), \dots, S(n)}(X|\hat{\alpha}(s_1), \dots, \hat{\alpha}(s_n)) < Var_{X|S(n)}(X|\tilde{\alpha}(s(n)))$$

Thus for the interval of $s_i \in [\mu_X - 3\sqrt{\sigma_X^2 + \sigma^2}, \mu_X + 3\sqrt{\sigma_X^2 + \sigma^2}]$, $\forall i$, there exists a threshold value \bar{r}_U such that when $r \geq \bar{r}_U$, we have $\hat{V}_n > V_n$.

Last, we demonstrate Proposition 6 by comparing the investors' utility under the public-

pay model \bar{V}_N with V_1 under the solicited rating system with a monopolistic CRA. Since

$$\begin{aligned}\bar{V}_N &= E_{X|S_{(1)}, \dots, S_{(n)}}(X|E(X|s_1), \dots, E(X|s_n)) - r \cdot \text{Var}_{X|S_{(1)}, \dots, S_{(n)}}(X|E(X|s_1), \dots, E(X|s_n)) \\ V_1 &= E_{X|S_i}(X|E(X|s_i)) - r \cdot \text{Var}_{X|S_i}(X|E(X|s_i))\end{aligned}$$

where $S_i \sim N(\mu_X, \sigma_X^2 + \sigma^2)$ and $X|S_i \sim N(\frac{\sigma^2 \cdot \mu_X + \sigma_X^2 \cdot s_i}{\sigma_X^2 + \sigma^2}, \frac{\sigma_X^2 \cdot \sigma^2}{\sigma_X^2 + \sigma^2})$, $\forall i$. Therefore, the variance of X conditional on n observations is smaller than that conditional on one CRA's observation s_i for $n > 1$:

$$\text{Var}_{X|S_{(1)}, \dots, S_{(n)}}(X|E(X|s_1), \dots, E(X|s_n)) = \sigma_X^2 \cdot \left(\frac{\sigma^2}{\sigma_X^2 + \sigma^2}\right)^n < \text{Var}_{X|S_i}(X|E(X|s_i)), \forall i$$

As we focus on the interval $s_i \in [\mu_X - 3\sqrt{\sigma_X^2 + \sigma^2}, \mu_X + 3\sqrt{\sigma_X^2 + \sigma^2}]$, $\forall i$, there exists a threshold value \bar{r}_p such that when $r \geq \bar{r}_p$, we have $\bar{V}_N > V_1$. **Q.E.D**